WORKING PAPERS

AN APPLICATION TO THE TICINO VALLEY PARK OF A MATHEMATICAL MODEL TO ANALYSE THE VISITORS BEHAVIOUR

C. S. Bertuglia, R. Tadei

ires ISTITUTO RICERCHE ECONOMICO - SOCIALI DEL PIEMONTE

Working Paper n. 4

AN APPLICATION TO THE TICINO VALLEY PARK
OF A MATHEMATICAL MODEL
TO ANALYSE THE VISITORS BEHAVIOUR

C. S. Bertuglia, R. Tadei WPn4

July 1981

To be presented at the XXI European Conference of the Regional Science Association, Barcelona, Spain, August 25-28, 1981

ABSTRACT

In this paper a Markov model to analyse the visitors behaviour in a country park is presented. The aim of the study is to find a 'good' organisation of the park to achieve a visitors distribution which is compatible with the protection of the natural environment.

An application of this model has been made at the Ticino Valley park in Piedmont, Italy. In this paper an interesting correlation analysis between the attraction factors of the park and its natural-physical and recreational features is presented. The results of this analysis may help the public authority in the park planning process.

Finally, some results of the use of the model to achieve a rebalancing of the disequilibrium caused by tourist pressure in the Ticino Valley park are outlined.

CONTENTS

1. Introduction

A natural park is a valuable resource, in that it represents an undisturbed natural environment which elsewhere has all too often been interfered with, and as such should be safeguarded. As the natural equilibrium is frequently very delicate and therefore easily upset, leading to a process of degradation, the park will also require protection.

In addition we can regard the park as a rare 'good', which should therefore be exploited, compatible with the restraints resulting from the above.

It follows that the problem is to determine a system of organisation which allows maximum use while at the same time safeguarding the natural environment from mis-use or over-use. This organisation will require that density limits of use for the zones of the park are respected.

In order to arrive at a system of organisation which fulfils these conditions, it is necessary to study the distribution and the behaviour of the park visitors.

In relation to the above, the following assumption have been made:

a. the visitors enter the park by means of a certain number of entry points. Their subsequent dispersal within the park will depend, on the one hand, upon the location of recreational opportunities and, on the other, upon impedence to movement between the entry point and these opportunities.

If all the visitors, having reached the opportunity or group of opportunities, remained there for their entire stay in the park, the distribution obtained from the above would be valid for the whole day. But this is not a realistic assumption, therefore it is supposed that:

b. the visitors, once distributed between the various opportunities of the park are redistributed at least once. This redistribution will depend firstly upon the location of the opportunities and secondly the impedences to movement among them.

Once a "good" configuration for the park has been established, we can see whether the policies necessary to achieve this are such that our goals are met, i.e.

- exploitation of the recreational potential
- avoidance of over-use which could lead to
  a) physical damage to the natural environment
  b) loss of attractiveness due to overcrowding.

To be able to do this we must have at our disposal a mathematical model of measuring the density of use by zone, generated by the above configuration, and, having identified any discrepancies between the desired and actual densities, modify the configuration until these imbalances are eliminated. The final configuration arrived at through this process will be the one adopted as park plan.

The flov-chart of the planning process described above is shown in figure 1.

Such a planning process has been applied at the Ticino Valley Park in Piedmont (Italy): the aim being to indicate to the public authority how it can and should intervene to achieve a rebalancing of the disequilibrium caused by tourist pressure and to provide a means of indicating to what extent its goals are being met.

2. A Markov model for customers behaviour in a country park

2.1. Preliminary definitions

Let consider a country park as a system formed by "states". We define state i of the park system S the couple

$$i = (x, h) \in S$$

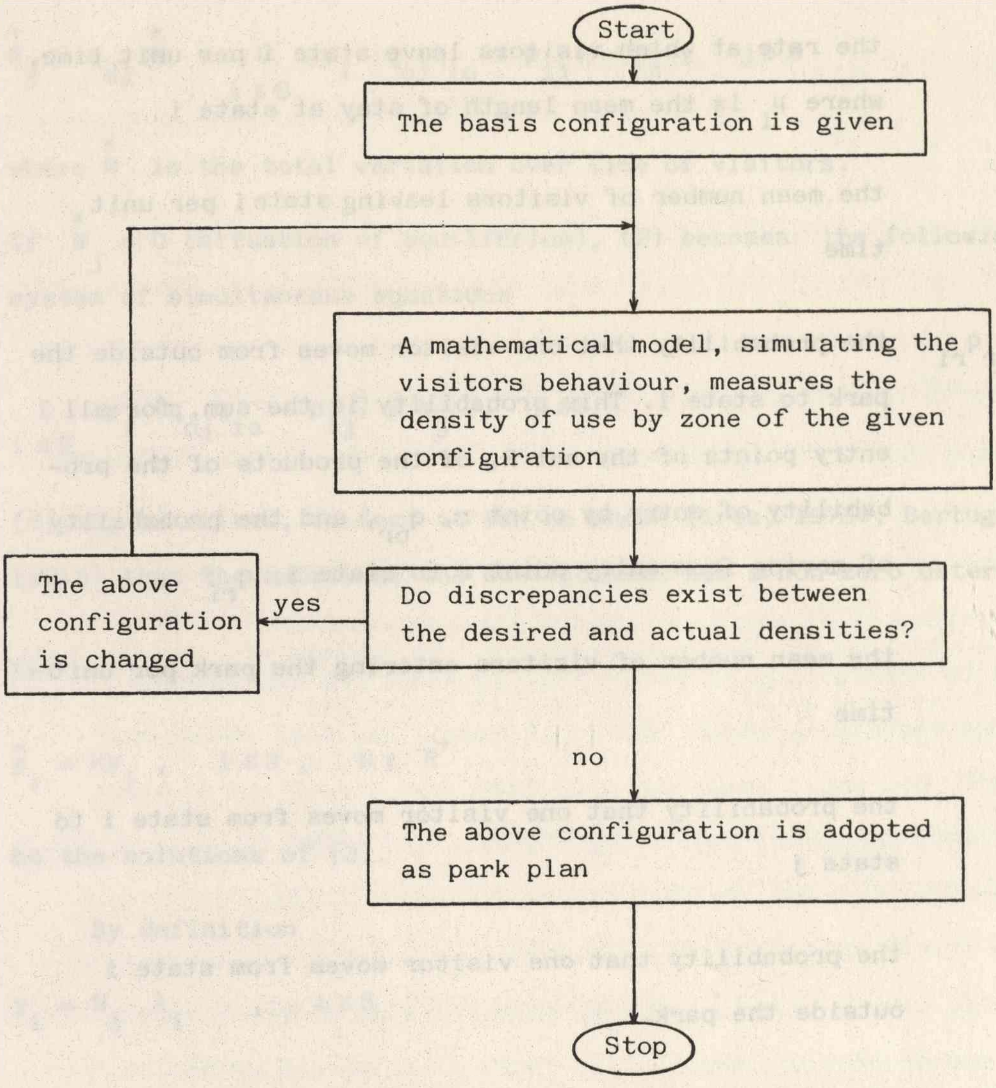

FIGURE 1. Flow-chart of the planning process

where $x \in X$ is one of the zones the park is divided into and $h \in H$ is one of the recreational activities practicable in the park.

The aim of the model is to calculate, analysing the visitors behaviour, the mean number of visitors present at each state $i \in S$.

To do this we must define

N the mean total number of park visitors

N_i the mean number of visitors at state i

$\lambda_i = 1/\mu_i$ the rate at which visitors leave state i per unit time, where μ_i is the mean length of stay at state i

$y_i = N_i \lambda_i$ the mean number of visitors leaving state i per unit time

$q_{oi} = \sum_{r \epsilon R} q_{or} q_{ri}$ the probability that one visitor moves from outside the park to state i. This probability is the sum, for all entry points of the set R, of the products of the probability of entry by point r, q_{or}, and the probability of moving from entry point r to state i, q_{ri}

s the mean number of visitors entering the park per unit time

p_{ij} the probability that one visitor moves from state i to state j

q_{io} the probability that one visitor moves from state i outside the park.

2.2. The model equations

We give the main equations of the model and no proof of them. For a wider discussion of the model equations and their proofs see Bertuglia and Tadei (1981a).

The variation over time of the number of visitors at state $j \epsilon S$ is

$$\dot{N}_j = s q_{oj} + \sum_{\substack{i \epsilon S \\ i \neq j}} y_i p_{ij} - y_j \quad , \quad j \epsilon S \tag{1}$$

It can be shown that (1) is equivalent to

$$\dot{N}_j = q_{oj}\dot{N} + \sum_{i \varepsilon S} y_i (q_{oj} q_{io} + p_{ij}) - y_j \quad , \quad j \varepsilon S \tag{2}$$

where $\dot{N}$ is the total variation over time of visitors.

If $\dot{N}_j = 0$ (situation of equilibrium), (2) becomes the following homogenous system of simultaneous equations

$$\sum_{i \varepsilon S} y_i (q_{oj} q_{io} + p_{ij}) = y_j \quad , \quad j \varepsilon S \tag{3}$$

(3) may be solved, in fact it can be shown (Ires, 1979b, Bertuglia and Tadei, 1981b) that the matrix of the coefficients has a non-zero determinant.

Let

$$\bar{y}_i = k y_i \quad , \quad i \varepsilon S \; , \quad k \varepsilon R^+ \tag{4}$$

be the solutions of (3).

By definition

$$y_i = N_i \lambda_i \quad , \quad i \varepsilon S \tag{5}$$

and the obvious constraint

$$\sum_{i \varepsilon S} N_i = N \tag{6}$$

holds.

So, from (4), (5), (6), we have

$$N_i = N (\bar{y}_i / \lambda_i) / (\sum_{i \varepsilon S} \bar{y}_i / \lambda_i) \quad , \quad i \varepsilon S \tag{7}$$

(7) is our model. The problems consists to find the solutions $\bar{y}_i$, $i \varepsilon S$ of the system (3). To do this we must know the quantities q_{io}, p_{ij}, q_{oj} (or q_{or} and q_{rj} , in fact, by definition, $q_{oy} = \sum_{r \varepsilon R} q_{or} q_{rj}$) , $i \varepsilon S$, $r \varepsilon R$.

Let us suppose that q_{io}, $i \in S$ are known.

To determine p_{ij}, q_{or}, q_{rj} $i, j \in S$, $r \in R$ we can use the entropy maximizing method.

This method is related to the information theory as developed by Shannon and Weaver (1949). Its use for statistical estimation and model building has been first proposed by Kullback (1959) and it has been applied to such diverse fields as economics (Bacharach, 1970), traffic analysis (Evans, 1970), geography and planning (Wilson, 1970, 1974), migrations (Willekens, Pór and Raquillet, 1981).

Since most applications deal with estimating flows among different states of a system, it is specially suited for our problem.

It might be interesting to notice that recently the equivalence between entropy maximizing models and logit models has been recognized (Williams, 1977, Brotchie, Lesse and Roy, 1979, Van Lierop and Nijkamp, 1979, Coelho 1980, Leonardi, 1981). Logit models are widely used as models of choice behaviour in human science (Domencich and McFadden, 1975, and many others). Therefore the entropy maximizing approach proposed here may be considered as a technique to estimate parameters in a set of linked logit models and to find p_{ij} and q_{ri}, $i, j \in S$, $r \in R$ we can write

$$\max E = - \sum_{i \in S} \sum_{j \in S} p_{ij} \ln p_{ij} - \sum_{i \in S} \sum_{r \in R} q_{ri} \ln q_{ri} \tag{8}$$

subject to the constraints:

$$\sum_{i \in S} p_{ij} = 1 - q_{io}, \qquad i \in S \tag{9}$$

$$\sum_{j \in S} p_{ij} c_{ij} = \bar{C}_i, \qquad i \in S \tag{10}$$

$$\sum_{j \in S} p_{ij} \ln w_j = L_i, \qquad i \in S \tag{11}$$

$$\sum_{i \in S} q_{ri} = 1 \qquad r \in R \tag{12}$$

$$\sum_{i \in S} q_{ri} c'_{ri} = \bar{C}'_r , \qquad r \in R \tag{13}$$

$$\sum_{i \in S} q_{ri} \ln w_i = L'_r , \qquad r \in R \tag{14}$$

where, given the states i = (x,h), j = (y,k) and the entry point r we have:

c_{ij}, c'_{ri} the travel time from zone x to zone y and from the entry point r to zone x

$\bar{C}_i$, $\bar{C}'_r$ the mean travel time from zone x to the other zones of the park and from the entry point r to the zones of the park

$\ln w_j$ a measure of the benefit obtained by the visitors at state j, see Wilson (1974), where w_j is a measure of the attraction of state j

L_i, L'_r a measure of the mean benefit obtained by the visitors at state i when they move to any of the other states and of the visitors coming from entry point r when they reach one of the states within the park.

Solving the non linear mathematical programming problem (8), we find

$$p_{ij} = (1 - q_{io}) w_j^{\eta_i} e^{-\beta_i c_{ij}} \Big/ \sum_{j \in S} w_j^{\eta_i} e^{-\beta_i c_{ij}} , \quad i, j \in S \tag{15}$$

and

$$q_{ri} = w_i^{\alpha_r} e^{-\gamma_r c'_{ri}} \Big/ \sum_{i \in S} w_i^{\alpha_r} e^{-\gamma_r c'_{ri}} , \quad r \in R, \ i \in S \tag{16}$$

where β_i, η_i, γ_r, α_r, $i \in S$, $r \in R$ are the Lagrange multipliers, to be calibrated, associated respectively with the constraints (10),(11),(13),(14). In an analogous way we can obtain q_{or}, $r \in R$. Now we are able to use the system (7) and the aim to find the distribution of visitors N_i, $i \in S$ has been achieved.

3. An application to the Ticino Valley park

3.1. The problem description

The model of section 2. is used to find a "good" organisation of the Ticino Valley park. This is a river country park on the border between two big Italian regions, Piedmont and Lombardy, and has a size of 6,250 a.. This park is used by daily tourism visitors to do the following leasure activities:

1. having a bath and taking the sun
2. boating
3. fishing
4. picnicking along the river
5. picnicking in the rest of the park
6. walking in the park.

The Ticino Valley park is a social park (i.e. it is used by tourists), but also a natural one. In fact, important zones from a botanic and natural point of view are found in it. These zones have to be protected and developed.

Then the problem is to find an equilibrium between the tourist exploitation of the park, so necessary because of the shortage of this kind of facilities in Italy, and the safeguarding of the important natural features of the park.

To have an idea of the tourist exploitation of the park, it is enough to think that in the peak hours 12,800 visitors are in the park, coming from Piedmont and Lombardy.

The Ticino Valley park, as a system, because of its physical features, may be shared into four indipendent systems (*).

(*) Two systems, forming a park system, are indipendent when the transit from one to the other implies going out of the park.

For example, the application of the model to the system 1. of figure 2 is discussed.

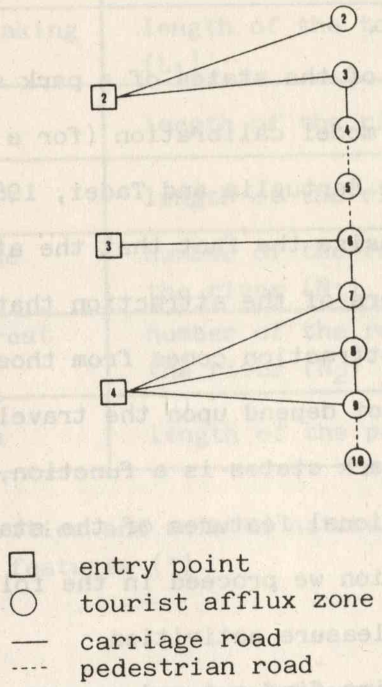

FIGURE 2. The system 1. of the Ticino Valley park

The system 1. is constituted by 3 entry points and 9 tourist afflux zones, in which the above leasure activities may be practised.

Recalling the definition of state of the park system (a state is a couple zone-leasure activity), in the system 1. we have 9 by 6 = 54 states. For each of these states we look for the maximum capacity, i.e. the maximum number of visitors present at the same time and avoiding any damage phenomenon of the natural environment.

We compare this number with the number of visitors actually present in the states. If these two numbers differ beyond a given tolerance, the necessary policies to modify the visitors distribution until these imbalances are eliminated are found by means of the mathematical model. These policies modify the travel times among the zones of the park and the attraction factors of the states. While it is clear how to modify the travel times, it is not so clear how to modify the attraction factors of the states. In the following section a way to solve this problem is presented.

3.2. The correlation analysis between the attraction factors and the natural-physical and recreational features of the park

The attraction factors of the states of a park system, w_i, $i \in S$ in the model, are an output of the model calibration (for a complete description of the model calibration see Bertuglia and Tadei, 1981a, Bertuglia, Gualco and Tadei, 1981a). We emphasize the fact that the attraction factors w_i, $i \in S$ may be seen as indicators of the attraction that each state exercises on the park visitors. This attraction comes from those elements (qualitative and quantitative) which do not depend upon the travel times. We may suppose that the attraction of the park states is a function, to find out, of the natural-physical and recreational features of the states.

To determine this function we proceed in the following way

1. consider each of the six leasure activities
2. for each of these activities find out and measure the natural-physical and recreational features we suppose necessary(and then attractive) to practise the activity itself
3. hypothesize a function that ties the values of the features of the point 2. to the values of the attraction factors output of the model calibration
4. verify, experimentally, by means of a correlation analysis the goodness of the hypothesis of the point 3. (*).

The six leasure activities and the natural-physical and recreational features we suppose necessary to practise them are shown in table 1.

(*) We observe that the point 3. is the delicate point of the above procedure, in fact neither applied nor theoretical previous studies to determine this kind of function do not exist.

Activities	Natural-physical and recreational features
1. having a bath and taking the sun	length of the tourist afflux zone (L_1)
2. boating	length of the river bank (L_2)
3. fishing	length of the river bank (L_2)
4. picnicking along the river	number of the refreshment places near the river (N_1)
5. picnicking in the rest of the park	number of the refreshment places far from the river (N_2)
6. walking in the park	length of the pedestrian roads (L_3)

TABLE 1. Leasure activities and related natural-physical and recreational features (*)

For each activity, the supposed relationships between the attraction factors $w = \{w_i, i \in S\}$ and the natural-physical and recreational features are shown in table 2.

Activities	Relationships between the attraction factors and the natural-physical and recreational features
1. having a bath and taking the sun	$w^1 = \alpha_1 \exp(\beta_1 L_1)$
2. boating	$w^2 = \alpha_2 + \beta_2 \ln L_2$
3. fishing	$w^3 = \alpha_3 + \beta_3 \ln L_2$
4. picnicking along the river	$w^4 = \alpha_4 \exp(\beta_4 N_1)$
5. picnicking in the rest of the park	$w^5 = \alpha_5 + \beta_5 \ln N_2$
6. walking in the park	$w^6 = \alpha_6 + \beta_6 \ln L_3$

TABLE 2. For each activity, relationship between the attraction factors and the natural-physical and recreational features

---

(*) The choice of these kinds of natural-physical and recreational features is also conditioned by the available data.

The parameters α_j, β_j, $j = 1, \ldots, 6$ in table 2 must be calibrated.

The equations in table 2 are of the following two kinds

a. $w^j = \alpha_j \exp(\beta_j y)$, $j = 1, \ldots, 6$

b. $w^j = \alpha_j + \beta_j \ln y$, $j = 1, \ldots, 6$

where y is the generic natural-physical and recreational feature.

The main feature of the equations a. is that the attraction factor w^j increases in an exponential way with y ('logit' model), while for the equations b., it increases in a logarithmic way.

We think that the attraction factor shape in the case a. fits very well the activities 1., having a bath and taking the sun, and 4., picnicking along the river. This fact may be explained because the Ticino Valley park is a river park and then the activities related to the water are exalted. Moreover for these activities we may suppose that the presence of a high number of customers is an attraction for new ones.

For the other activities the attraction factor shape in the case b. seems to fit very well. For istance, for the activity 3.,fishing, it seems logical that when the length of the river bank, L_2, increases the related value of the attraction factor does not increase in a directly or more than directly proportional way. A similar consideration may be made for the activities 2., 5. and 6.. We observe that, although the activities 2., boating, and 3., fishing, are related to the water, the attraction factor shape differs from that of the activities 1. and 4. This may be justified because for the activities 2. and 3. the presence of a high number of customers is not an attraction but even an obstacle for new ones. The shape of the equations of table 2 and the values of the correlation coefficients of the correlation analysis to find the parameters α_j and β_j, $j = 1, \ldots, 6$ are given in figures 3, 4, 5, 6, 7, 8.

We note that the curve in figure 3 starts from a point of the positive semi-axis of ordinates. Il follows that, even when the tourist afflux zone does not exit (i.e. $L_1 = 0$), the visitors use the park to practise the

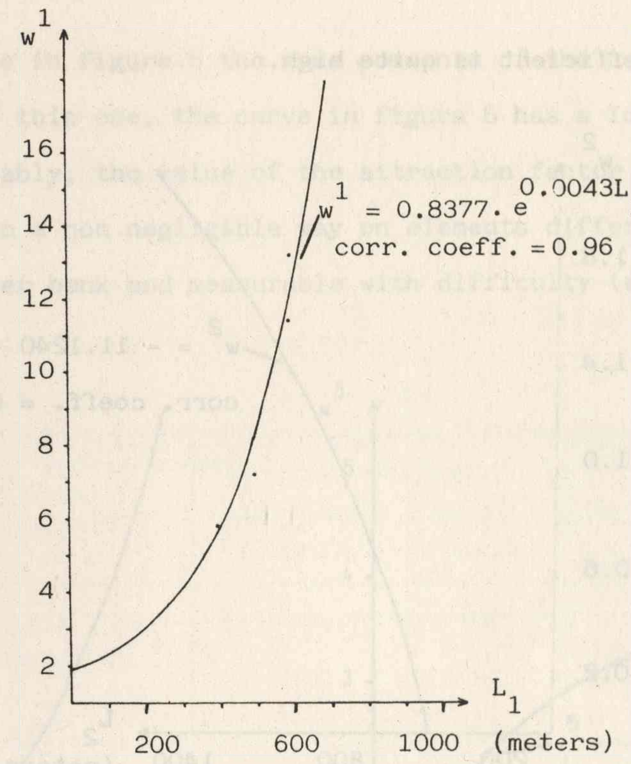

FIGURE 3. Relationship between the attraction factor w^1 and the length of the tourist afflux zone L_1 for the activity having a bath and taking the sun

activity 1.. In this case they do not have a bath but only take the sun and this agrees with the actual visitors behaviour.

As shown in figure 3 the value of the correlation coefficient is quite high (*).

We note that the curve in figure 4 starts from a point of the positive semi-axis of abscissas. It follows that, if the river bank does not exceed a given length, the visitors do not practise the activity boating and this agrees with the observed visitors behaviour. In this case too the value of

---

(*) Tentatively, for each leasure activity we hypothesized functions differing from those of table 2 (linear and logarithmic functions for the activities 1. and 4. and linear and exponential functions for the activities 2., 3., 5. and 6.). In any case we obtained values of the correlation coefficients lower than those obtained with the initial hypothesis.

the correlation coefficient is quite high.

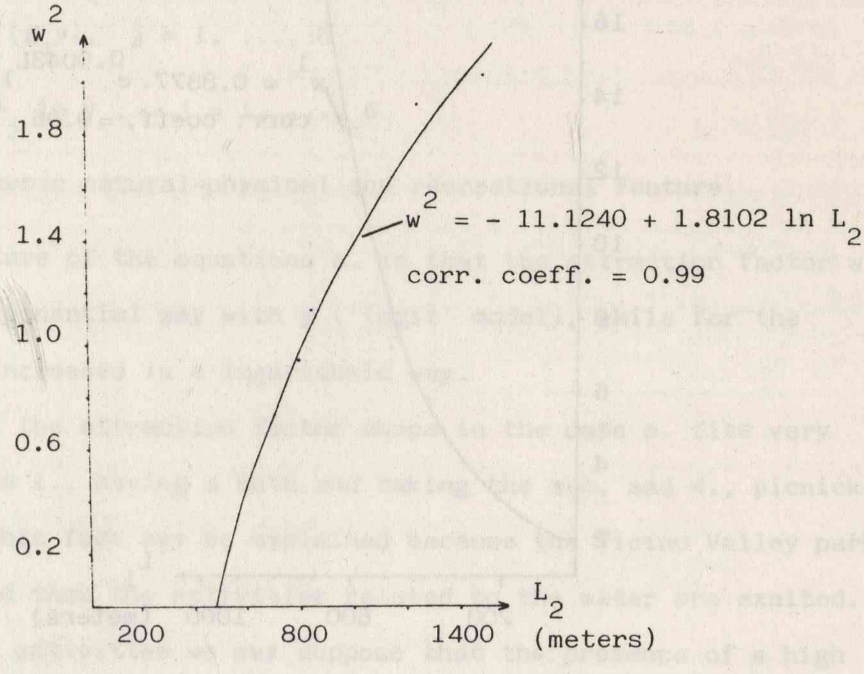

FIGURE 4. Relationship between the attraction factor w^2 and the length of the river bank L_2 for the activity boating

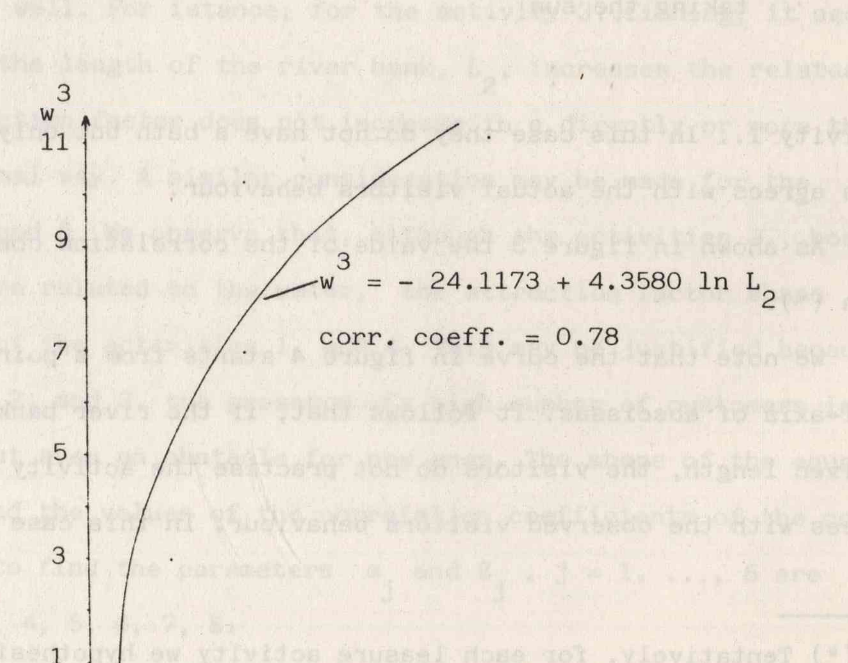

FIGURE 5. Relationship between the attraction factor w^3 and the length of the river bank L_2 for the activity fishing

For the curve in figure 5 the same comments of the curve in figure 4 hold. In spite of this one, the curve in figura 5 has a low correlation coefficient. Probably, the value of the attraction factor for the activity fishing depends in a non negligible way on elements differing from the length of the river bank and measurable with difficulty (e.g. the fish availability!).

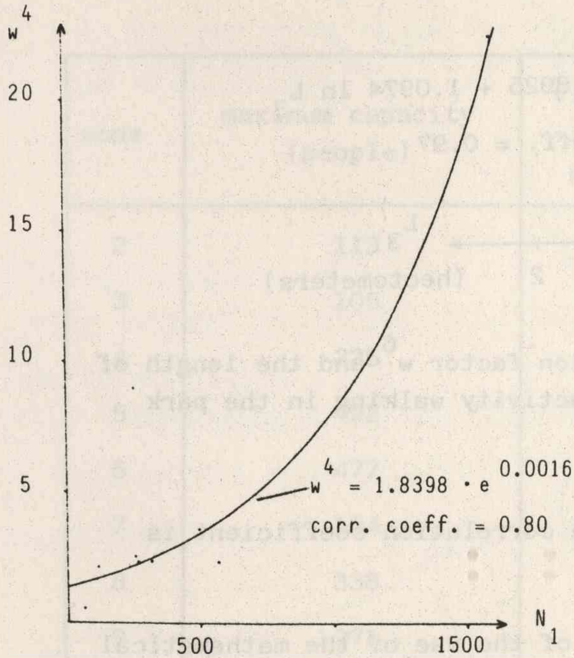

FIGURE 6. Relationship between the attraction factor w^4 and the number of the refreshment places near the river N_1 for the activity picnicking along the river

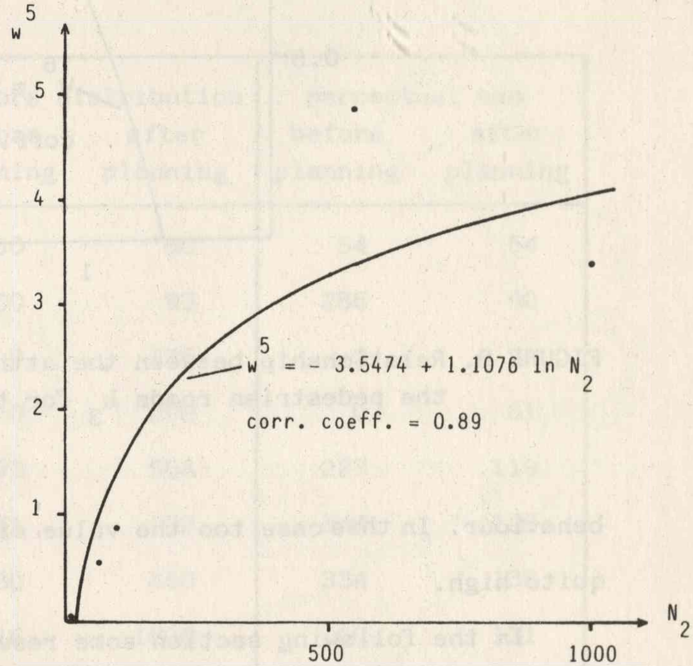

FIGURE 7. Relationship between the attraction factor w^5 and the number of the refreshment places far from the river N_2 for the activity picnicking in the rest of the park

For a comment of the different shape of the attraction factor in relation to analogous variables as the number of the refreshment places near the river and far from the river (figures 6 and 7) see the comment made for the table 2.

We note that the curve in figure 8 states from a point of the positive semi-axis of abscissas. It follows that, if the length of the pedestrian roads does not exceed a given length, the visitors do not practise the activity walking in the park and this agrees with the observed visitors

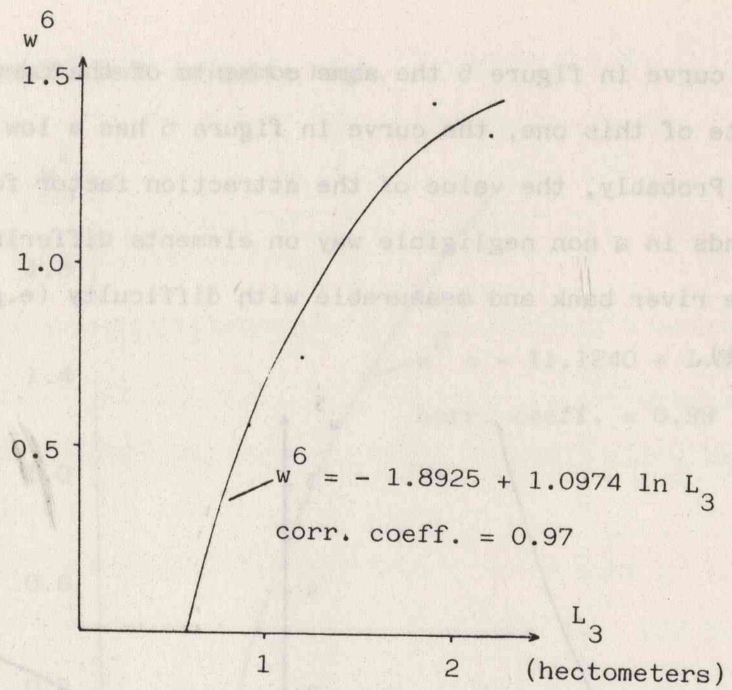

FIGURE 8. Relationship between the attraction factor w^6 and the length of the pedestrian roads L_3 for the activity walking in the park

behaviour. In this case too the value of the correlation coefficient is quite high.

In the following section some results of the use of the mathematical model of section 2 and of the correlation analysis of section 3.2. are presented.

3.3. The use of the model: some results

We recall that the mathematical model is used to find out the necessary policies to modify the visitors distribution in order to eliminate possible imbalances between the maximum capacity and the actual number of visitors of the park zones. We assume that these imbalances exit when the actual number of visitors is either lower than the maximum capacity minus 50% or higher than the maximum capacity plus 50%. The ± 50% tolerance is necessary because of the weak theoretical basis of the maximum capacity definition.

In figure 9 the visitors distribution in the system 1. of the Ticino Valley park before and after planning is shown.

We can easily note, see table 3 too, that before planning we have

- two zones in equilibrium (2 and 9)
- four zones with over-use (3, 6, 7 and 8)
- three zones with mis-use (4, 5 and 10).

zone	maximum capacity (people)	visitors distribution before planning	visitors distribution after planning	percentual use before planning	percentual use after planning
2	113	60	60	54	54
3	105	300	95	286	90
4	225	0	175	0	78
5	492	0	298	0	61
6	477	1083	568	227	119
7	134	461	172	344	128
8	338	1130	460	334	136
9	1371	816	1800	60	131
10	321	0	222	0	69

TABLE 3. The maximum capacity and the actual use of the park zones before and after planning

To obtain a 'better' visitor distribution that eliminates the above imbalances, we try to modify the attraction factors of the park zones. To do this we use the curves of the correlation analysis between the attraction factors and the natural-physical and recreational features of the park of section 3.2.

In other terms we suppose a set of policies which modify L_1, L_2, L_3, N_1 and N_2, then we calculate the new values of the attraction factors and using the mathematical model we obtain a new visitors distribution. We carry

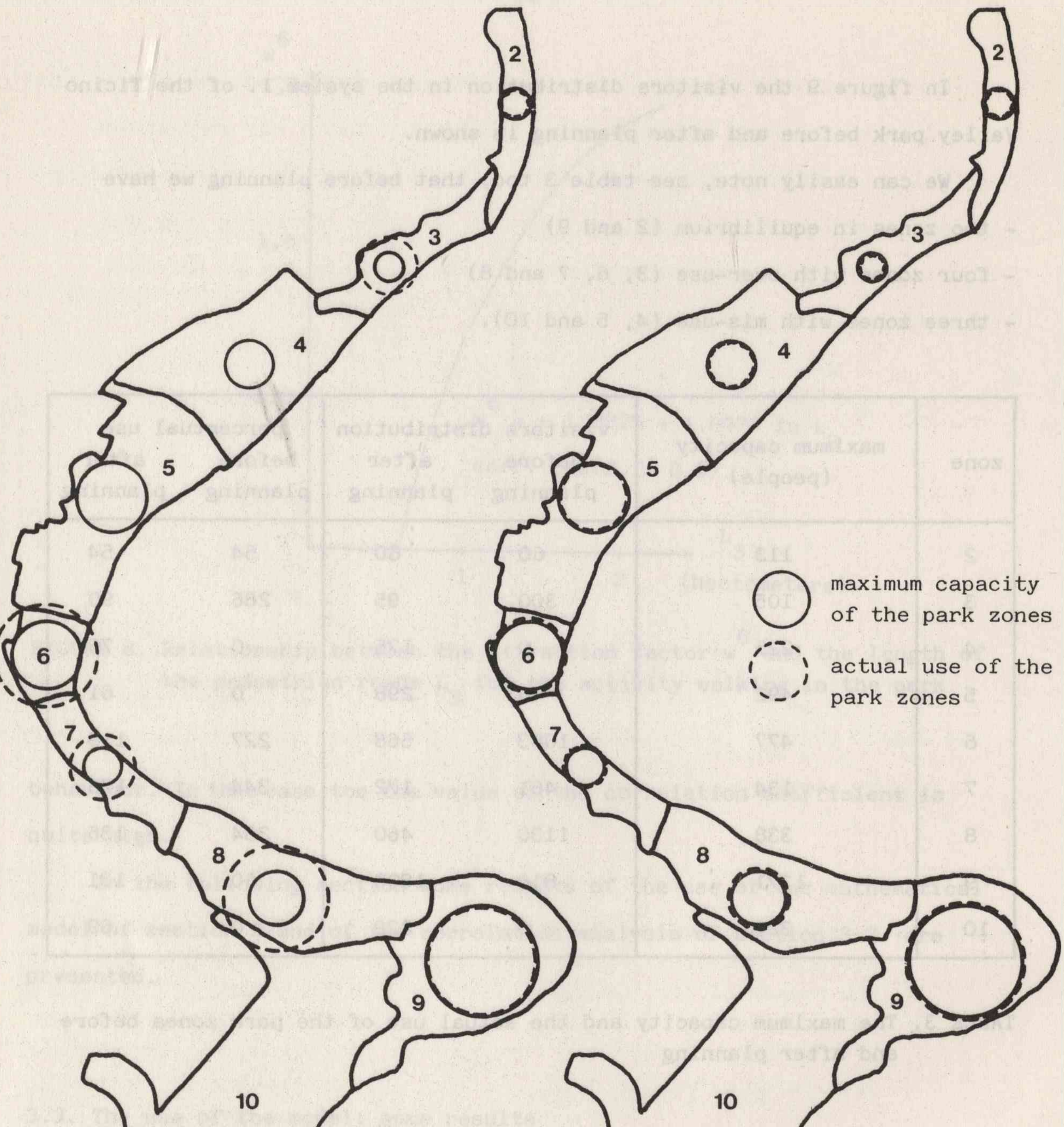

FIGURE 9. The visitors distribution in the system 1. of the Ticino Valley park before and after planning

on until a 'good' visitors distribution is found.

In this specific case, policies modifying the length of the tourist afflux zone L_1, the number of the refreshment places near the river N_1 and the length of the pedestrian roads L_3 have been enough to obtain the after planning visitors distribution shown in figure 9. We can see that in this case each zone has met the desired equilibrium. For the quantification of the used policies and for their implementation see Bertuglia, Gualco and Tadei (1981b).

4. Concluding comments and issues for future research

The main aim of the paper has been to show the effectiveness of the use of this mathematical model to solve problems arising from the interaction between a natural resource, as a country park, and its customers, as the park visitors.

A strand of future research is the construction of a more compact indicator of the results of the implemented policies. Moreover we must consider the fact that these policies, modifying the attraction factors, may increase the tourist demand of the park. Further theoretical studies on the definition of the maximum capacity of the park zones are also required.

References

Bacharach, M., "Biproportional Matrices and Input-Output Analysis", Cambridge University Press, London (1970).

Batty, M., "Urban Modelling", Cambridge University Press, Cambridge, 175-207 (1976).

Baxter, R.S., "Computer and Statistical Techniques for Planners", Mathuen & Co. Ltd,.London, 309-331 (1976).

Bertuglia, C.S., "Linee per l'organizzazione del parco sociale del Ticino", Giardini, Pisa (1975).

Bertuglia, C.S., Gualco, I. and Tadei, R., "Applicazione al parco naturale della valle del Ticino di un modello per l'analisi del comportamento degli utenti: la calibrazione del modello", Atti delle giornate AIRO, Torino, 2,321-344 (1981a).

Bertuglia, C.S., Gualco, I. and Tadei, R., "Applicazione al parco naturale della valle del Ticino di un modello per l'analisi del comportamento degli utenti: l'uso del modello", Atti delle giornate AIRO, Torino, 2, 345-362 (1981b).

Bertuglia, C.S., Leonardi, G. and Tadei, R., "The optimal management of natural recreational resources: a mathematical model", Environment & Planning, 12, 69-83 (1980).

Bertuglia, C.S., Ramasco, E. and Tadei, R., "Contributo alla formazione di una metodologia per l'organizzazione di un sistema di parchi regionali", Atti delle giornate AIRO, Parma, 1, 408-421 (1977).

Bertuglia, C.S., and Tadei, R., "A Stochastic Model for the Use of a Country Park", Ecological Modelling, forthcoming (1981a).

Bertuglia, C.S., and Tadei, R., "Metodologie per la pianificazione dei parchi regionali", Working paper 2, Ires (1981b).

Brotchie, J.F., Lesse, P.F. and Roy, J.R., "Entropy, utility, and planning models", Sistemi Urbani, 3, 33-53 (1979).

Coelho, J.D., "Optimizaçao, interacçao espacial e teoria do comportamento", Nota n° 5, Centro de Estatistica e Aplicaçoes, Dept. Matemática Aplicada, Faculdade de Ciências de Lisboa (1980).

Domencich, T. and McFadden, D., "Urban Travel Demand: A Behavioral Analysis", North Holland Publishing Company, Amsterdam (1975).

Evans, A.W., "Some properties of trip distribution methods", Transportation Research, 4, 19-36 (1970).

Ires - Istituto Ricerche Economico-Sociali del Piemonte, "Politiche di Comunicazione in Valle di Susa", Franco Angeli Editore, Milano, 90-290 (1979a).

Ires - Istituto Ricerche Economico-Sociali del Piemonte, "Programma di lavoro per la formazione del piano del parco del Ticino", Torino (1979b).

Kullback, S., "Information Theory and Statistics", J.Wiley & Sons, New York (1959).

Leonardi, G., "The use of random-utility theory in building location-allocation models", WP-81-28, IIASA, Laxenburg (1981).

Shannon, C.E. and Weaver, W., "The Mathematical Theory of Communication", University of Illinois Press, Urbana, Ill. (1949).

Shechter, M. and Lucas, R., "Simulation of Recreational Use for Park and Wilderness Management", Johns Hopkins, Baltimore (1978).

Van Lierop, W. and Nijkamp, P., "A utility framework for interaction models", Sistemi Urbani, 1, 41-64 (1979).

Willekens, F. Pór, A. and Raquillet, R., "Entropy, multiproportional, and quadratic techniques for inferring patterns of migration from aggregate data", RR-81-6, IIASA, Laxenburg (1981).

Williams, H.C.W.L., "On the formation of travel demand models and economic evaluation measures of user benefit", Environment and Planning A, 9, 203-205 (1977).

Wilson, A.G., "Entropy in Urban and Regional Modelling", Pion, London, (1970).

Wilson, A.G., "Urban and Regional Models in Geography and Planning", J. Wiley & Sons, London (1974).

References

Bacharach, M., "Biproportional Matrices and Input-Output Analysis", Cambridge [illegible].

Batty, M., "Urban Modelling", Cambridge University Press, Cambridge, 175-207, [illegible].

Baxter, R.S., "Computer and Statistical Techniques for Planners", Methuen [illegible].

Bertuglia, C.S., "Linee per l'organizzazione del parco naturale del Ticino" [illegible].

Bertuglia, C.S., Gualco, I. and Tadei, R., "Applicazione al parco naturale [illegible] gli utenti: la calibrazione [illegible]", Torino, 2, 321-344 (1981a).

Bertuglia, C.S., Gualco, I. and Tadei, R., "Applicazione al parco naturale della valle del Ticino di un modello per l'analisi del comportamento de gli utenti: l'uso del modello", Atti delle giornate AIRO, Torino, 2, 345-362 (1981b).

Bertuglia, C.S., Leonardi, G. and Tadei, R., "The optimal management of na tural recreational resources: a mathematical model", Environment & Planning, 12, 65-83 (1980).

Bertuglia, C.S., Romano, S. and Tadei, R., "Contributo alla formazione di una metodologia per l'organizzazione di un sistema di parchi regionali", Atti delle giornate AIRO, Parma, 1, 408-421 (1977).

Bertuglia, C.S., and Tadei, R., "A Stochastic Model for the Use of a Country Park", Ecological Modelling, forthcoming (1982).

Bertuglia, C.S., and Tadei, R., "Metodologie per la pianificazione dei parchi regionali", Working paper 2, Ires (1981b).

Brotchie, J.F., Lesse, P.F. and Roy, J.R., "Entropy, utility, and planning models", Sistemi Urbani, 3, 33-55 (1979).

Coelho, J.D., "Optimização, interacção espacial e teoria do comportamento", Nota nº 5, Centro de Estatística e Aplicações, Dept. Matemática Aplicada, Faculdade de Ciências de Lisboa (1980).

Domencich, T. and McFadden, D., "Urban Travel Demand: A Behavioral Analysis", North Holland Publishing Company, Amsterdam (1975).

Evans, A.W., "Some properties of trip distribution methods", Transportation Research, 4, 19-36 (1970).

ires ISTITUTO RICERCHE ECONOMICO - SOCIALI DEL PIEMONTE
VIA BOGINO 21 10123 TORINO